

# Determination of $\alpha_s$ at NLO\*+NNLL from a global fit of the low- $z$ parton-to-hadron fragmentation functions in $e^+e^-$ and DIS collisions

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**Abstract.** The QCD coupling  $\alpha_s$  is determined from a combined analysis of experimental  $e^+e^-$  and  $e^+p$  jet data confronted to theoretical predictions of the energy evolution of the parton-to-hadron fragmentation functions (FFs) moments –multiplicity, peak, width, skewness– at low fractional hadron momentum  $z$ . The impact of approximate next-to-leading order (NLO\*) corrections plus next-to-next-to-leading log (NNLL) resummations, compared to previous LO+NNL calculations, is discussed. A global fit of the full set of existing data, amounting to 360 FF moments at collision energies  $\sqrt{s} \approx 1\text{--}200$  GeV, results in  $\alpha_s(m_Z^2) = 0.1189^{+0.0025}_{-0.0014}$  at the Z mass.

## 1 Introduction

As a consequence of asymptotic freedom in quantum chromodynamics (QCD), the strong coupling  $\alpha_s$  decreases logarithmically with increasing energy scale  $Q$ . At leading order,  $\alpha_s \propto \ln^{-1}(Q^2/\Lambda_{\text{QCD}}^2)$  starting from a value  $\Lambda_{\text{QCD}} \approx 0.2$  GeV where the perturbatively-defined coupling diverges and the relevant degrees of freedom are not quarks and gluons (collectively called partons) but colour-neutral hadrons. Theoretical calculations of the parton energy evolution, through gluon radiation and quark-antiquark splitting, usually start at scales well above  $Q_0 \approx 1$  GeV, i.e. for  $Q \gg Q_0 \geq \Lambda_{\text{QCD}}$ , such that perturbation theory can be safely applied as a convergent expansion in powers of  $\alpha_s$ , while softer large-distance ( $Q \leq Q_0$ ) phenomena, including the final hadronization, are encoded into experimentally-measured parton-to-hadron fragmentation functions (FFs). FFs can be interpreted as the probability for a parton to produce a hadron which carries a fraction  $z$  of the total longitudinal momentum of the jet.

At large  $z \gtrsim 0.1$ , one can extract a value of  $\alpha_s$  from the scaling violations of the FFs at next-to-leading order (NLO) accuracy via the comparison of inclusive cross-sections for hadron production measured in high-energy particle collisions with theoretical predictions [1]. The obtained  $\alpha_s(m_Z^2) = 0.1170 \pm 0.0073$  value at the Z mass pole in such approaches is consistent with the current (NNLO) world average  $\alpha_s(m_Z^2) = 0.1185 \pm 0.0006$  [2], derived from a variety of measurements at  $e^+e^-$ , deep-inelastic scattering (DIS)  $e^\pm, \nu$ -p, and hadron-hadron colliders. The bulk of

hadron production from jets is, however, concentrated at low  $z \lesssim 0.1$  where parton evolution is dominated by singularities due to soft and collinear gluon bremsstrahlung [3] approaching  $\Lambda_{\text{QCD}} \approx 0.2$  GeV. Indeed, due to colour coherence and gluon-radiation interference inside a parton shower, partons with intermediate energies ( $E_h \propto E_{\text{jet}}^{0.3}$ ) multiply most effectively in QCD cascades, leading to a final hadron spectrum peaked at low  $z$ , with a typical “hump-backed plateau” (HBP) shape as a function of  $\xi = \ln(1/z)$ . The HBP shape of the single-inclusive distribution of hadrons in jets can be parametrized, without any loss of generality, as a distorted Gaussian (DG) which depends on the original energy of the parton,  $Y \approx \ln E/Q_0$ , evolved down to a shower cutoff scale  $\lambda = \ln(Q_0/\Lambda_{\text{QCD}})$ :

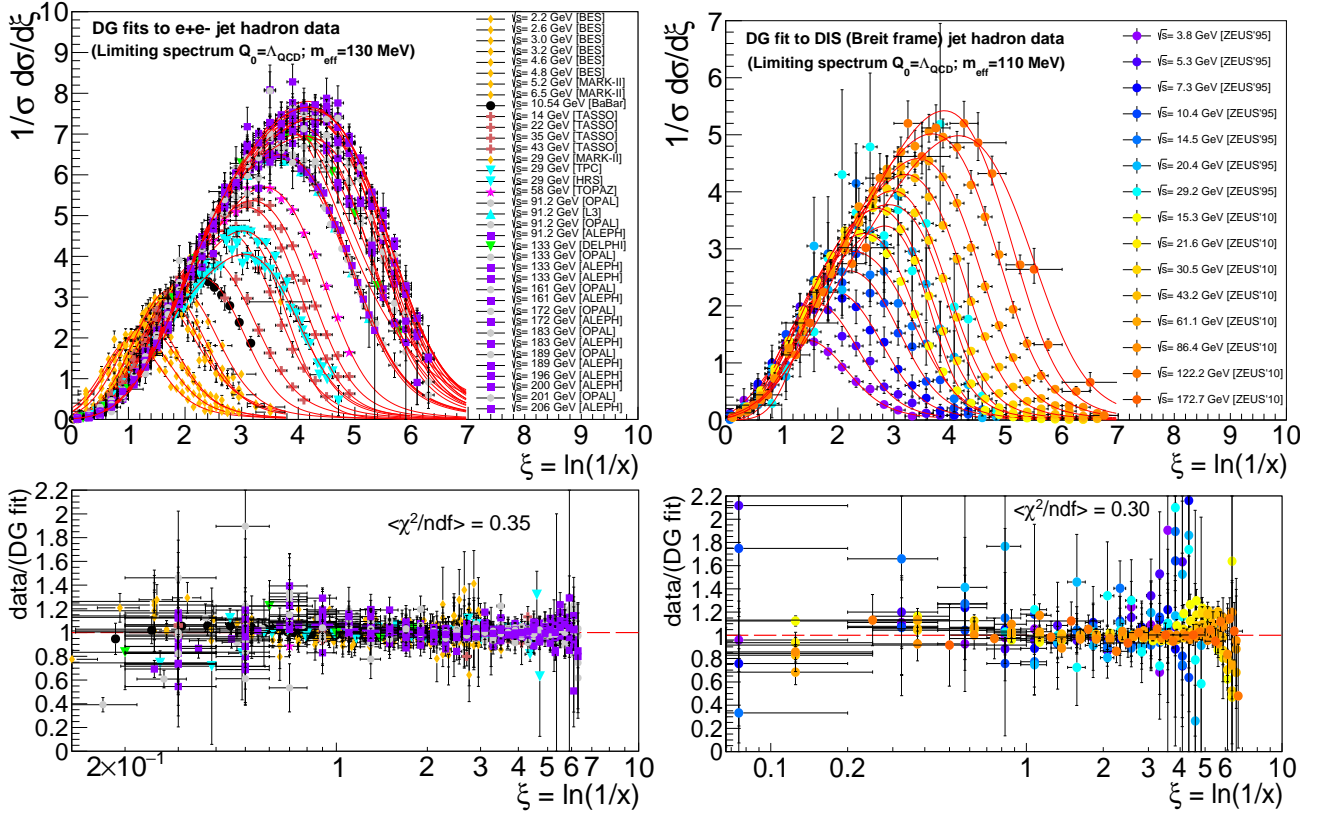
$$D^+(\xi, Y, \lambda) = \frac{\mathcal{N}}{(\sigma \sqrt{2\pi})} \cdot e^{\left[\frac{1}{8}k - \frac{1}{2}s\delta - \frac{1}{4}(2+k)\delta^2 + \frac{1}{6}s\delta^3 + \frac{1}{24}k\delta^4\right]}, \quad (1)$$

where  $\delta = (\xi - \bar{\xi})/\sigma$ , with moments:  $\mathcal{N}$  (hadron multiplicity inside the jet),  $\bar{\xi}$  (DG peak position),  $\sigma$  (DG width),  $s$  (DG skewness), and  $k$  (DG kurtosis). Figure 1 shows the FFs measured in  $e^+e^-$  (37 datasets, left) and DIS (15 datasets, right) fitted to the DG expression (1).

Thanks to kinematical constraints on the parton branching process, such as exact angular ordering in the  $s$ -channel, it is possible to study the evolution of quark and gluon FFs also at low  $z$  (i.e. at high  $\xi$ ) via renormalized equations that resum all singularities at leading-log (LLA) accuracy and beyond (Modified Leading Logarithmic Approximation MLLA [4], and next-to-MLLA [5]). Attempts to extract  $\alpha_s$  from the low- $z$  (i.e. high- $\xi$ ) moments of the FFs were carried out in the past at MLLA accuracy (see e.g. [6]). However, these older calculations

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**Figure 1.** Top: Charged-hadron distributions in jets as a function of  $\xi = \ln(1/z)$  measured in  $e^+e^-$  at  $\sqrt{s} \approx 2\text{--}200$  GeV (left) and  $e^\pm, \nu\text{-p}$  (Breit frame, scaled up by  $\times 2$  to account for the full hemisphere) at  $\sqrt{s} \approx 1\text{--}180$  GeV (right), individually fitted to the distorted Gaussian, Eq. (1), with the hadron mass corrections ( $m_{\text{eff}} = 120 \pm 20$  MeV) quoted. Bottom: Ratio of the measured FFs to each DG fit.

relied on a number of simplifying assumptions: (i) ad hoc cuts in the experimental distributions, (ii) simple fits in a restricted FF range, (iii) number of quark-flavours fixed to  $N_f = 3$ , (iv) use of only one or two FF moments (Gaussian approximation), and (v) LO expression for  $\alpha_s$ . As a result, inconclusive values of  $\Lambda_{\text{QCD}} \approx 80\text{--}600$  MeV were reported (see e.g. [7]), although the latest studies of the energy evolution of the first FF moment alone (i.e. of the hadron multiplicity in jets) have yielded more precise  $\alpha_s$  results consistent with the world average [8]. Recently, we have presented a novel extraction of  $\alpha_s$  based on the fit of the energy evolution of the first four moments of the low- $z$  parton-to-hadron FFs including resummations of next-to-next-to-leading logarithms (NNLL or NMLLA) complemented with NLO\* running-coupling corrections<sup>1</sup>, where more terms have been consistently added to the perturbative expansion compared to previous works, and where a large set of experimental FF data has been systematically analysed for the first time [9–11]. Thus, by fitting the experimental single-inclusive hadron distribution for jets at various energies to the DG parametrization (1), one can determine  $\alpha_s$  from the corresponding energy-dependence of its fitted moments. Since the current world-average  $\alpha_s$  uncertainty is of order  $\pm 0.5\%$ —although more conservative estimates place it at the  $\pm 1\%$  level [12], making of  $\alpha_s$  the least precisely-known of all fundamental

couplings in nature—having at hand extra independent approaches to determine  $\alpha_s$ , with experimental and theoretical uncertainties different than those of the methods currently used, is an obvious advantage.

In this work, we present first a more detailed study of the relative role of the higher order (NLO\*+NNLL) corrections included in our analytical expressions for the multiplicity, maximum peak position, width, skewness, and kurtosis of the FFs, compared to the LO+NLL (or MLLA) expressions obtained in the past. In a second stage, we do a combined study of  $e^+e^-$  and DIS jet FF data, including a few (older) datasets not incorporated in our previous analyses [9–11], and we carry out a *single* global fit of all DG moments, rather than independent ones for  $e^+e^-$  and DIS collisions, in order to extract a more precise value of  $\alpha_s$ .

## 2 Theoretical framework

The parton-to-hadron FF,  $D_{i \rightarrow h}(z, Q)$ , encodes the probability that parton  $i$  fragments into a hadron  $h$  carrying a fraction  $z$  of the parent parton's momentum. FFs for gluon and different flavors of quark-initiated jets can be computed from DGLAP evolution equations [13–15] in perturbation theory. As for the Schrödinger equation in quantum mechanics, the system of equations can be written as an evolution *Hamiltonian* which mixes gluon and (anti)quark states expressed in terms of DGLAP splitting

<sup>1</sup>The asterisk in the term 'NLO\*' stands for 'approximate NLO' as there are missing corrections in the splitting functions.

functions for the branchings  $g \rightarrow gg$ ,  $q(\bar{q}) \rightarrow qg(\bar{q})$  and  $g \rightarrow q\bar{q}$ , where  $g$ ,  $q$  and  $\bar{q}$  label a gluon, a quark and an antiquark respectively. Analytical solutions can be obtained by using a Mellin transform over the convolution product of the regularized splitting functions and the FFs with respect to  $\xi$ . For soft partons, the shift in  $\xi$  is reabsorbed into the exponential such that the energy radiated by the parton  $\omega$  can be replaced by  $\omega \rightarrow \Omega = \omega + \partial/\partial Y$  (where  $Y$  is related to the (log) of the energy of the initial parton), resulting in a final expansion in half-powers of  $\alpha_s$ . In order to incorporate  $O(\alpha_s^{3/2})$  contributions, going beyond the  $O(\alpha_s)$  terms obtained in older approaches, the matrix elements of the evolution *Hamiltonian* should be expanded up to terms  $\propto \Omega$ , followed by its diagonalisation, which results into two eigenvalues  $P_{\pm\pm}(\Omega)$  in the new  $\mathcal{D}^\pm(\Omega, Q)$  basis. This procedure leads to the following equation for the eigenvector  $\mathcal{D}^+$  [9],

$$\left(\omega + \frac{\partial}{\partial Y}\right) \frac{\partial}{\partial Y} \mathcal{D}^+(\omega, Y, \lambda) = \left[1 - \frac{a_1}{4N_c} \left(\omega + \frac{\partial}{\partial Y}\right) + a_2 \left(\omega + \frac{\partial}{\partial Y}\right)^2\right] 4N_c \frac{\alpha_s}{2\pi} \mathcal{D}^+(\omega, Y, \lambda) \quad (2)$$

which is the one that provides a Gaussian-like shape for the distribution, while  $\mathcal{D}^-$  vanishes asymptotically. As explained in [9],  $a_1$  and  $a_2$  are hard constants depending on the number of active flavors  $N_f$  and on the  $C_F$  and  $N_c$  Casimirs of the fundamental and adjoint representation of the SU(3) color group respectively; and  $\lambda = \ln(Q_0/\Lambda_{\text{QCD}})$  is the hadronization parameter at which the shower stops. The terms  $\propto a_1$  and  $a_2$  provide respectively NLL and NNLL corrections. Equation (2) is solved by using the two-loop expression of  $\alpha_s$ :

$$\alpha_s(q^2) = \frac{4\pi}{\beta_0 \ln q^2} \left[1 - \frac{2\beta_1}{\beta_0^2} \frac{\ln \ln q^2}{\ln q^2}\right], \text{ with } q^2 = \frac{Q^2}{\Lambda_{\text{QCD}}^2}, \quad (3)$$

and  $\beta_{0,1}$  are the first two coefficients of the perturbative expansion of the  $\beta$ -function through the renormalisation group equation [16].

The solution of Eq. (2) can be written in the compact form:

$$\mathcal{D}^+(\omega, Y, \lambda) = E_+(\omega, \alpha_s(Y + \lambda)) \mathcal{D}^+(\omega, \lambda), \quad (4)$$

with the evolution *Hamiltonian* rewritten in terms of the anomalous dimension  $\gamma(\omega, \alpha_s)$  as:

$$E_+(\omega, \alpha_s(Y + \lambda)) = \exp \left[ \int_0^Y dy \gamma(\omega, \alpha_s(y + \lambda)) \right], \quad (5)$$

with  $y = Y - \xi$ , which describes the parton jet evolution from its initial virtuality  $Q$  to the lowest possible energy scale  $Q_0$ , at which the parton-to-hadron transition occurs. Inserting Eq. (5) into (2), the resulting equation for  $\gamma(\omega, \alpha_s)$  can be solved iteratively. Its solution at NLO\*+NNLL accuracy reads:

$$\gamma_\omega^{\text{NLO}^*+\text{NNLL}} = \gamma_\omega^{\text{MLLA}} + \frac{\gamma_0^4}{16N_c^2} \left\{ a_1^2 \frac{\gamma_0^2}{(\omega^2 + 4\gamma_0^2)^{3/2}} \right.$$

$$+ \frac{a_1\beta_0}{2} \left( \frac{1}{\sqrt{\omega^2 + 4\gamma_0^2}} - \frac{\omega^3}{(\omega^2 + 4\gamma_0^2)^2} \right) + \beta_0^2 \left( \frac{2\gamma_0^2}{(\omega^2 + 4\gamma_0^2)^{3/2}} - \frac{5\gamma_0^4}{(\omega^2 + 4\gamma_0^2)^{5/2}} \right) - 4N_c \frac{\beta_1}{\beta_0} \frac{\ln 2(Y + \lambda)}{\sqrt{\omega^2 + 4\gamma_0^2}} \left. \right\} + \frac{1}{4} a_2 \gamma_0^2 \left[ \frac{\omega}{(\omega^2 + 4\gamma_0^2)^{1/4}} + (\omega^2 + 4\gamma_0^2)^{1/4} \right]^2, \quad (6)$$

where  $\gamma_\omega^{\text{MLLA}}$  is the MLLA anomalous dimension computed first by Fong & Webber [17] and  $\gamma_0 \sim \sqrt{\alpha_s}$  is the anomalous dimension obtained in the double logarithmic approximation (DLA) [18]. The MLLA anomalous dimension resums double soft-collinear leading logarithms (DLA, of order  $O(\sqrt{\alpha_s})$ ) and hard-collinear next-to-leading logs (single logarithms, of order  $O(\alpha_s)$ ) which partially restore energy conservation and account for LO running coupling effects. Terms proportional to  $\beta_1$  and  $a_2$  are corrections computed for the first time in our NMLLA+NLO\* framework. In the expression proportional to  $\gamma_0^4$ , the terms  $\propto a_1^2, a_1\beta_0, \beta_0^2$  mix NNLL contributions obtained iteratively, the  $\propto a_2$  term resums next-to-next-to-leading logarithms which improve energy conservation at each branching vertex of the parton shower and, finally, the  $\propto \beta_1$  term accounts for NLO running coupling effects. The overall new correction is  $O(\alpha_s^{3/2})$  as can be checked from the power counting in Eq. (6), knowing that  $\omega \sim O(\alpha_s^{-1/2})$ .

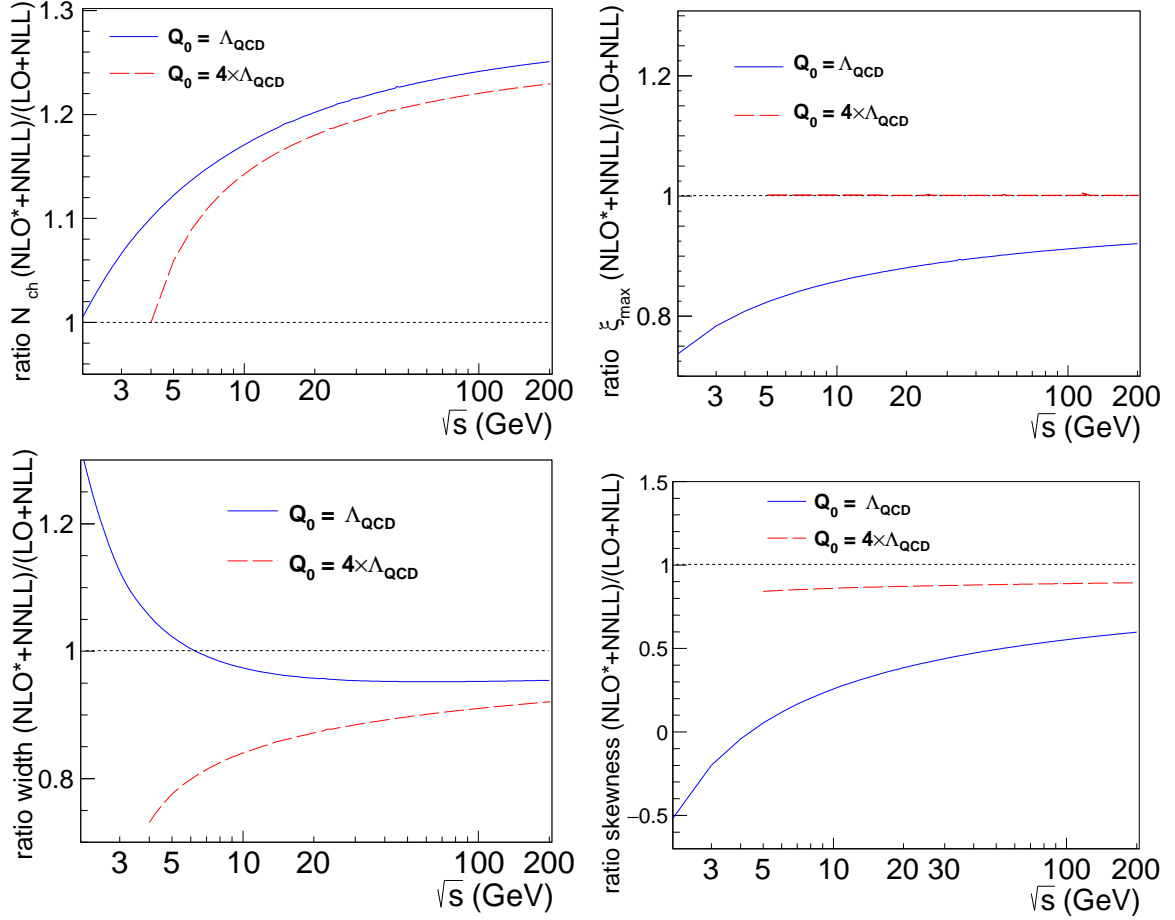
Replacing the Mellin transform of Eq. (1) into Eq. (4), the DG moments can be obtained at NLO\*+NNLL accuracy from the anomalous dimension (6) by comparing both sides of the resulting equation, via

$$K_{n \geq 0}(Y, \lambda) = \int_0^Y dy \left( -\frac{\partial}{\partial \omega} \right)^n \gamma_\omega(\alpha_s(y + \lambda)) \Big|_{\omega=0}, \quad (7)$$

with

$$\mathcal{N} = K_0, \quad \bar{\xi} = K_1, \quad \sigma = \sqrt{K_2}, \quad s = \frac{K_3}{\sigma^3}, \quad k = \frac{K_4}{\sigma^4}. \quad (8)$$

The full expressions of the energy evolution of the moments (8) have been derived as a function of  $Y = \ln(E/\Lambda_{\text{QCD}})$  (for an initial parton energy  $E$ ) and  $\lambda$  for  $N_f = 3, 4, 5$  in [9]. The resulting formulae for the energy evolution of the moments depend on  $\Lambda_{\text{QCD}}$  as *single* free parameter. Particularly simple expressions are obtained in the limiting-spectrum case ( $\lambda = 0$ , i.e. for  $Q_0 = \Lambda_{\text{QCD}}$ ) motivated by the “local parton hadron duality” hypothesis for infrared-safe observables which states that the distribution of partons in jets are simply renormalized in the hadronization process without changing their HBP shape. In Fig. 2 we display the ratios (NLO\*+NNLL)/(LO+NLL) of the moments so as to shed more light on the size of the new corrections of Eq. (6) compared to older LO+NLL results. We plot the ratios for the limiting-spectrum case and for a scale  $\lambda = 1.4$  (i.e. for  $Q_0 = 4\Lambda_{\text{QCD}} \approx 1 \text{ GeV}$ ).



**Figure 2.** Ratios of the theoretical predictions for the energy evolution of the moments of the parton-to-hadron FFs: multiplicity (top-left), peak (top-right), width (bottom-left) and skewness (bottom-right) at NLO\*+NNLL over LO+NLL for two values of the shower cutoff  $Q_0 = \Lambda_{\text{QCD}}$  (solid curve) and  $Q_0 = 4 \cdot \Lambda_{\text{QCD}} \approx 1 \text{ GeV}$  (dashed curve).

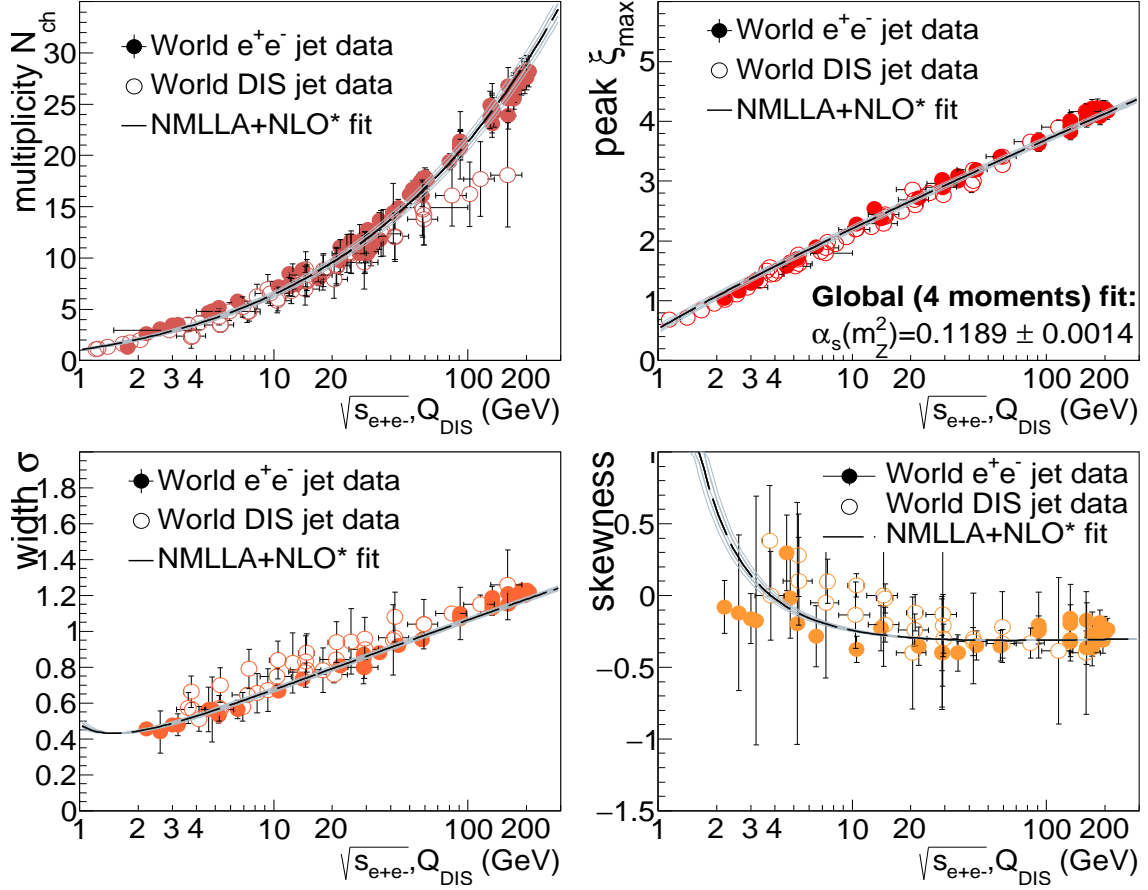
Various observations are worth pointing out. First, the NLO\*+NNLL corrections are quite sizable for all DG moments in the limiting spectrum but smaller for larger values of the shower energy cutoff. This is not surprising since, as  $\lambda$  increases, the convergence of the perturbative expansion is improved, higher order corrections decrease much faster and the (NLO\*+NNLL)/(LO+NLL) ratios approach unity in each case (solid curves), as observed. Second, focusing in the simplest limiting-spectrum case, we see that the impact of higher-order corrections is different in size and in sign for the different FF moments. On the one hand, the predictions for the multiplicity are larger by up to a 25% in the NLO\*+NNLL framework compared to the LO+NLL predictions. On the other, the DG peak and skewness are smaller at NLO\*+NNLL by about 10–20% and more than 50% respectively, whereas the DG width evolution varies by  $\pm(10\text{--}20)\%$  depending on the jet energy. For a fixed jet energy, higher hadron multiplicity, DG peak and width translate into comparatively lower values of the associated  $\Lambda_{\text{QCD}}$ , whereas higher values of the DG skewness reflect larger  $\Lambda_{\text{QCD}}$ . Those results highlight the importance of properly accounting for higher-order contributions in any combined analysis of the energy evolution

of the FF moments. Work is in progress to include the full-NLO (and beyond) corrections of the evolution of the FF moments [19].

### 3 Extraction of $\alpha_s$

The procedure of extraction of  $\alpha_s$  from the experimental data consists in a two-step process. First, we collect all the existing parton-to-hadron FFs measured so far and fit them to the DG expression (1) in order to obtain four FF moments at each jet energy. Finite hadron-mass effects in the DG fit have been accounted for through a rescaling of the theoretical (massless) parton momenta with an effective mass<sup>2</sup>  $m_{\text{eff}}$  as discussed in Ref. [9]. Next, we carry out a combined fit of the four moments as a function of the original parton energy (which in the case of  $e^+e^-$  collisions corresponds to half the centre of mass energy  $\sqrt{s}/2$  and, for DIS, to the invariant four-momentum transfer  $Q$ ) to our NMLLA+NLO\* predictions leaving  $\Lambda_{\text{QCD}}$  as a free

<sup>2</sup>Since the measured FFs are for massive hadrons and the calculations assume massless partons/hadrons (for which  $\xi_p = \xi_E$ ), the expression (1) for  $\xi$  needs to be modified by introducing an effective mass,  $E = \sqrt{p^2 + m_{\text{eff}}^2}$ , plus the corresponding Jacobian determinant correction.



**Figure 3.** Global NLO\*+NNLL fit of the energy evolution of the moments (charged hadron multiplicity, peak, width and skewness) of the jet FFs measured in  $e^+e^-$  collisions at  $\sqrt{s_{e^+e^-}} \approx 2\text{--}200$  GeV (solid circles) and  $e^\pm, \nu$ -p collisions at  $Q_{\text{DIS}} \approx 4\text{--}180$  GeV (open circles). The extracted value of  $\alpha_s(m_Z^2)$  is quoted in top-right panel.

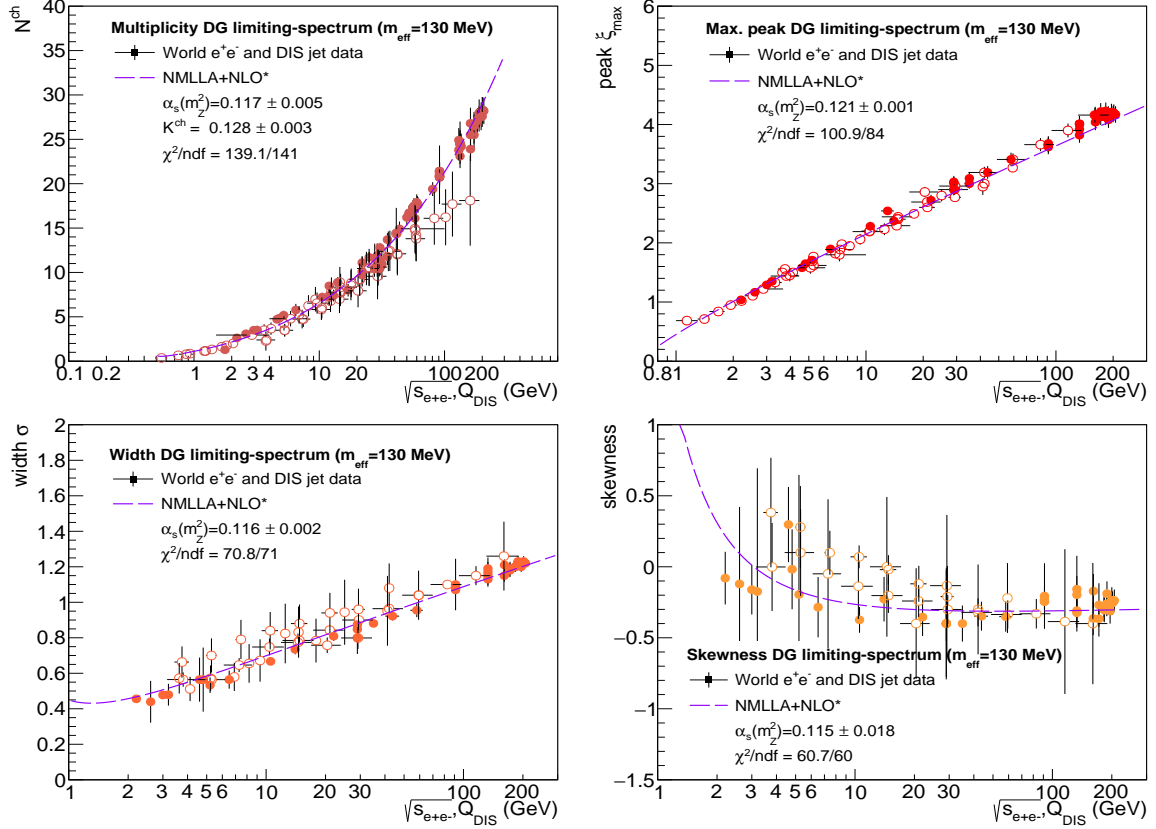
parameter in the fit. From the extracted  $\Lambda_{\text{QCD}}$ , we obtain the value of the QCD coupling at the Z pole,  $\alpha_s(m_Z^2)$ , using the two-loop running Eq. (3) for  $N_f = 5$  quark flavours.

### 3.1 Data sets and fits

Our analysis includes 37 measurements of FFs in  $e^+e^-$  collisions covering the range of c.m. energies  $\sqrt{s} \approx 2\text{--}200$  GeV from the following experiments: BES at  $\sqrt{s} = 2\text{--}5$  GeV [20]; BaBar<sup>3</sup> at  $\sqrt{s} = 10.54$  GeV [21]; MARK-II at  $\sqrt{s} = 5.2, 6.5$  and 29 GeV [22]; TASSO at  $\sqrt{s} = 14\text{--}44$  GeV [23, 24]; TPC/Two-Gamma at  $\sqrt{s} = 29$  GeV [25]; HRS at  $\sqrt{s} = 29$  GeV [26]; TOPAZ at  $\sqrt{s} = 58$  GeV [27]; ALEPH [28], L3 [29] and OPAL [6, 30] at  $\sqrt{s} = 91.2$  GeV; ALEPH [31, 32], DELPHI [33] and OPAL [34] at  $\sqrt{s} = 133$  GeV; and ALEPH [32] and OPAL [35–37] in the range  $\sqrt{s} = 161\text{--}202$  GeV. The analysis presented here extends our previous study [9] with 5 new datasets [21, 22, 26]. The total number of FF points is about 1000 (Fig. 1 left) and the systematic and statistical uncertainties of each single-hadron spectrum have been added in quadrature. Beyond the results of our DG fits, we add also other FF

moments which have been directly measured in  $e^+e^-$  collisions (not already included in the FF fits above) such as: 41  $N_{\text{ch}}$  values in the range  $\sqrt{s} = 12\text{--}161$  GeV compiled in [38], 3  $N_{\text{ch}}$  and  $\xi_{\text{max}}$  values measured by the JADE collaboration at  $\sqrt{s} = 12, 30, 35$  GeV [39], plus the average charged multiplicity measured in the world data [2] of hadronic decays of the Z boson ( $\langle N_{\text{ch}} \rangle = 20.76 \pm 0.16$ ), the W boson ( $\langle N_{\text{ch}} \rangle = 19.39 \pm 0.08$ ), and the  $\tau$  lepton ( $\langle N_{\text{ch}} \rangle = 1.314 \pm 0.002$ ) corresponding to  $\sqrt{s} = m_{Z,W,\tau} \approx 91.2, 80.4, 1.77$  GeV respectively. The total final number of FF moments extracted from the world  $e^+e^-$  jet data amounts to 200. In the case of DIS collisions, our analysis fits first the single-hadron distributions (amounting to about 250 individual data points, see Fig. 1 right) measured by ZEUS [40] in the current hemisphere of the Breit (or “brick wall”) frame where the incoming quark scatters off the photon and returns along the same axis. In addition, we include also into our global fit the 55 direct measurements of FF moments (mostly multiplicity, peak, and width) from H1 [41] and ZEUS [42] experiments in  $e^\pm$ -p at HERA, and NOMAD ( $\nu$ -N scattering) [43], covering the range of four-momentum transfers  $Q \approx 1\text{--}180$  GeV. The total final number of FF moments extracted from the DIS jet data is 160 and, thus, the combined global fit of  $e^+e^-$  and  $e^\pm, \nu$ -p moments amounts to 360 data points.

<sup>3</sup>The individual distributions measured for prompt pions, kaons and (anti)protons have been added into a single charged-particle distribution.



**Figure 4.** Individual NLO\*+NNLL fits of the energy dependence of the moments of the parton-to-hadron FFs measured in  $e^+e^-$  and DIS collisions: multiplicity (top-left), peak (top-right), width (bottom-left) and skewness (bottom-right); after  $\chi^2$ -reweighting of some DIS data (see text). The extracted values of  $\alpha_s(m_Z^2)$  (and associated goodness-of-fit  $\chi^2/\text{ndof}$ ) are quoted for each fit.

The combined fit of the energy-dependencies of the multiplicity, peak, width and skewness of the experimental FFs to our NLO\*+NNLL predictions for the evolution of the limiting spectrum FF moments, leaving  $\Lambda_{\text{QCD}}$  as free parameter, is shown in Fig. 3. The fit includes also corrections for each moment to account for the increasing number of flavours,  $N_f = 3, 4, 5$ , at the corresponding heavy-quark production thresholds:  $E_{\text{jet}} > m_{\text{charm, bottom}} \approx 1.3, 4.2$  GeV. The global fit, obtained using the MINUIT2 package (with the MIGRAD minimizer, although alternative algorithms give identical results) [44] implemented in ROOT, yields a QCD coupling strength at the Z mass pole of  $\alpha_s(m_Z^2) = 0.1189 \pm 0.0014$ . The values obtained from separate fits of our previous (smaller) sets of  $e^+e^-$  and DIS data are:  $\alpha_s(m_Z^2) = 0.1195 \pm 0.0022$  [9] and  $0.119 \pm 0.010$  [10], respectively. In the range of experimental jet energies considered,  $E_{\text{jet}} \approx 1\text{--}100$  GeV, the average scale at which  $\alpha_s$  is effectively evaluated in our approach is given by the geometric mean between the energy of the original parton and that at the end of the shower evolution, i.e.  $\langle Q \rangle = \sqrt{E_{\text{jet}} \cdot Q_0} \approx 0.6\text{--}2$  GeV in the limiting spectrum case.

### 3.2 Systematic uncertainties

Our extracted value of  $\alpha_s(m_Z^2) = 0.1189 \pm 0.0014$  has a relative uncertainty of about 1.2% (without theoretical scale uncertainties which are discussed below), which is a very competitive value compared to other existing  $\alpha_s$  determinations [2]. The  $\alpha_s(m_Z^2)$  uncertainties have been obtained through the “ $\chi^2$  averaging” method [2] as explained in [11]: We fit first the energy-dependence for each individual moment to its corresponding theoretical prediction (Fig. 4) and if the goodness-of-fit  $\chi^2$  is larger than the number of degrees of freedom (ndof), then the data points of the corresponding moment are enlarged by a common factor such that  $\chi^2/\text{ndof}$  equals unity. As a matter of fact, such an error enlargement has been applied only to two DIS moments:  $N_{\text{ch}}$  (which requires an overall +20% increase in its uncertainties) and width  $\sigma$  (+10% increase), which show a larger scatter in their central values compared to the  $e^+e^-$  measurements. In particular, as can be seen from the top-left panel of Fig. 4, the hadron multiplicities measured in DIS jets are systematically  $\sim 20\%$  smaller (especially at the highest energies) than those measured in  $e^+e^-$  collisions [9], a fact pointing maybe to limitations in the FF measurement only in half (current Breit)  $e^+p$  hemisphere. The  $\chi^2$ -averaging method takes into account in a well defined manner possible correlations among the evolutions



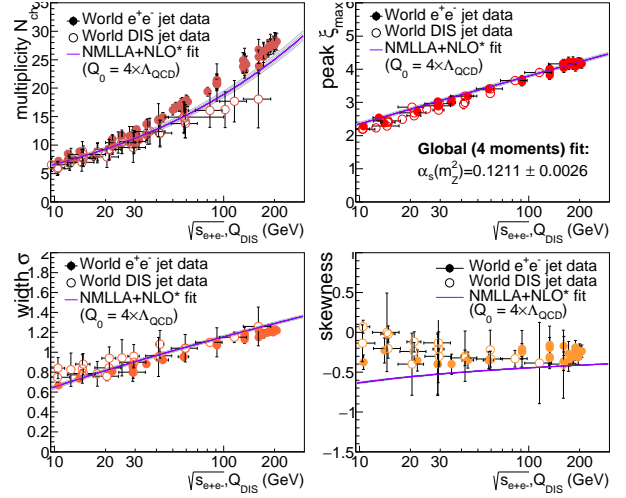
of the four FF moments, as well as any missing extra systematic uncertainties. Our detailed assessment of the experimental uncertainties [11] indicates that such an error assignment covers perfectly well the range of  $\alpha_s$  variations induced by the existing sources of uncertainty: (i) finite hadron-mass effects corrected through an  $m_{\text{eff}}$  factor introduced in Eq. (1), and (ii) the use of data-sets with slightly different definitions of final charged hadrons (including, or not, a fraction of secondary hadrons from weak  $K_s^0$  and  $\Lambda$  decays). Both effects have been estimated in [11] by scrutinizing the high-precision prompt and inclusive hadron FFs measured by the BaBar experiment [21]. We find that our FF analysis is robust with respect to hadronization and other experimental uncertainties. Indeed,

(i) The fit uncertainties for all extracted moments cover well the variations due to suitable choices of effective hadron masses in the range  $m_{\text{eff}} = 0\text{--}0.2$  GeV. The fits shown in Fig. 1 have been obtained for effective masses that result in the best goodness-of-fit ( $m_{\text{eff}} = 130$  MeV for  $e^+e^-$ , and 110 MeV for DIS), but conservative  $\pm 50\%$  variations from these default values yield consistent  $\alpha_s$  extractions.

(ii) The DG moments obtained from the FFs for prompt (primary) and inclusive (primary+decay) charged hadrons are all consistent within their associated uncertainties except, as expected, for the total multiplicity  $N_{\text{ch}}$  which is a factor of  $(9 \pm 1)\%$  smaller for the primary-hadrons FFs. Also, the analysis shows that older DIS extractions of the FF width using a simple Gaussian function overestimate their values by 20% compared to the more realistic DG fits<sup>4</sup>.

The last source of systematic uncertainty is of purely theoretical nature and it is associated with missing fixed-order terms in our truncation of the  $\alpha_s$  expansion at approximate NLO accuracy. Although our  $\alpha_s$  determination relies on a theoretically framework that resums soft and collinear logs down to  $Q_0 = \Lambda_{\text{QCD}}$ , and thus it is much more robust with respect to hadronization corrections than other methods which treat such effects as extra non-perturbative uncertainties, the state-of-the-art  $\alpha_s$  determinations have one-level of higher (NNLO) accuracy [2]. In order to estimate the size of theoretical scale uncertainties associated with missing higher-order terms, we have redone the analysis for a different energy scale  $\lambda$  at which the parton evolution is stopped. Using  $\lambda = 1.4$  (i.e.  $Q_0 = 4 \cdot \Lambda_{\text{QCD}} \approx 1$  GeV) and limiting the energy evolution fits to jets energies in the range  $E = 10\text{--}200$  GeV, so as to leave enough room for parton evolution avoiding data with energies too close to the shower cutoff at 1 GeV, we obtain  $\alpha_s(m_Z^2) = 0.1211 \pm 0.0026$  (Fig. 5). We see that the data-theory agreement is good for all FF moments except for the skewness which shows a better fit with the default limiting-spectrum ansatz. The QCD coupling value obtained stopping the parton evolution of the

FFs at 1 GeV is consistent with that determined in the limiting spectrum case,  $\alpha_s(m_Z^2) = 0.1189 \pm 0.0014$ , although larger by  $+0.0022$ . We conservatively assign this difference as a (positive) source of systematic error associated with the scale uncertainty of our calculations. Adding in quadrature this asymmetric error to the previously determined  $\pm 0.0014$  systematics uncertainty, results in the final quantitative result of our study:  $\alpha_s(m_Z^2) = 0.1189^{+0.0025}_{-0.0014}$ .



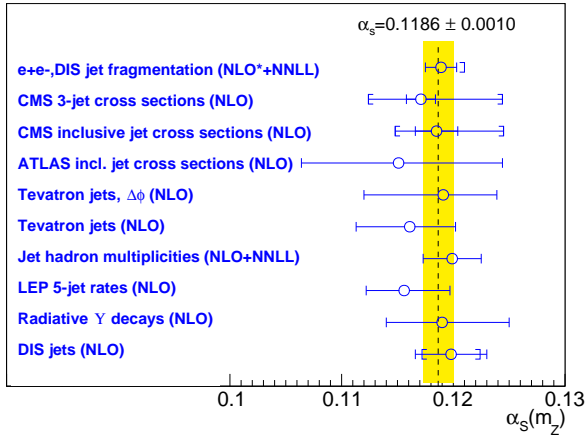
**Figure 5.** Energy evolution of the moments (charged hadron multiplicity, peak, width and skewness) of the jet FFs measured in  $e^+e^-$  and DIS collisions at  $\sqrt{s} \approx 10\text{--}200$  GeV, fitted to the NLO\*+NNLL predictions evaluated at a scale  $\lambda = 1.4$  (i.e.  $Q_0 = 4 \cdot \Lambda_{\text{QCD}} \approx 1$  GeV).

In Fig. 6 we compare our final  $\alpha_s(m_Z^2)$  value to all other existing results at NLO accuracy extracted from the latest PDG compilation [2] plus the most recent jet cross sections results from the CMS [45] and ATLAS [46] data. Our result is the most precise of all approaches while having a totally different set of experimental and theoretical uncertainties. A simple weighted average of all these NLO values yields:  $\alpha_s(m_Z^2) = 0.1186 \pm 0.0010$ , in perfect agreement with the current (NNLO) world-average.

## 4 Summary

The QCD coupling has been determined at NLO\*+NNLL accuracy from an analysis of the energy evolution of the first four moments (multiplicity, peak, width, skewness) of the parton-to-hadron fragmentation functions measured in  $e^+e^-$  and deep-inelastic collisions. A global fit of 360 FF moments in the energy range  $\sqrt{s} \approx 1\text{--}200$  GeV, yields  $\alpha_s(m_Z^2) = 0.1189^{+0.0025}_{-0.0014}$  at the Z mass, in perfect agreement with the current world-average of  $\alpha_s(m_Z^2) = 0.1185 \pm 0.0006$  (obtained at NNLO accuracy). The role of higher order corrections determined in this framework for the first time –namely approximate next-to-leading order (NLO\*) fixed-order effects on  $\alpha_s$  plus next-to-next-to-leading log (NNLL) resummations– has been assessed by comparing them to older results at LO+NLL accuracy, highlighting their importance for a

<sup>4</sup>Note that the original FFs are not available in some of the oldest DIS measurements [41] and their measured Gaussian widths have been included into our global energy-evolution fit with such a correction factor applied.



**Figure 6.** Summary of NLO  $\alpha_s$  determinations using different methods. The error “brackets” (if present) indicate the theoretical uncertainties of each extraction. The dashed line and shaded (yellow) band indicate their weighted average (listed also on the top).

quantitative analysis of the FF moments. We have studied in detail the systematic uncertainties associated with our  $\alpha_s$  extraction, finding that the fit uncertainties obtained through a  $\chi^2$ -averaging method fully cover the range of  $\alpha_s(m_Z^2)$  variations driven by hadronization (finite hadron-mass) corrections, different experimental final-hadron definitions, as well as the overall fit procedure. An additional theoretical-scale uncertainty of  $+0.0022$  has been obtained by redoing the fit of the FF moments using a different shower cutoff value (i.e. relaxing the limiting-spectrum criterion). The relative uncertainty of our extracted  $\alpha_s(m_Z^2)$  value,  $(-1.2\%, +2.1\%)$  is very competitive with respect to the other methods used so far to determine the QCD coupling. Work is in progress to include full-NLO (and beyond) corrections [19] which will further reduce the overall uncertainty (through a possible better data-theory agreement in the fits and/or a reduced theoretical scale error). The methodology presented here provides a novel high-precision approach for the determination of the QCD coupling strength complementary to other existing jet-based methods –such as jet shapes, and yields and ratios of N-jet production cross sections in  $e^+e^-$ , DIS and p-p collisions– and can be used to reduce the overall final uncertainty of the least well known interaction coupling in nature.

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